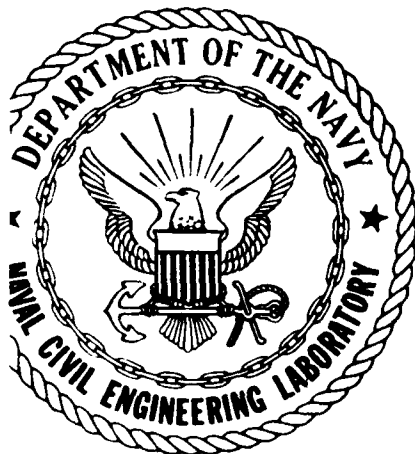MICROCOPY RESOLUTION TEST CHART

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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California

REVIEW OF PRELIMINARY ANALYSIS TECHNIQUES
FOR TENSION STRUCTURES

February 1984

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in²	square inches	6.5	square centimeters	cm²
ft²	square feet	0.09	square meters	m²
yd²	square yards	0.8	square meters	m²
mi²	square miles	2.6	square kilometers	km²
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft³	cubic feet	0.03	cubic meters	m³
yd³	cubic yards	0.76	cubic meters	m³
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

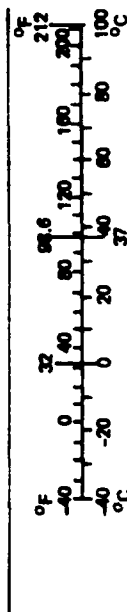## Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm²	square centimeters	0.16	square inches	in²
m²	square meters	1.2	square yards	yd²
km²	square kilometers	0.4	square miles	mi²
ha	hectares (10,000 m²)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m³	cubic meters	36	cubic feet	ft³
m³	cubic meters	1.3	cubic yards	yd³
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

°F
-40   0   32   98.6   212
-40 -20 0 20 40 60 80 100 120 140 160 180 200
-40 -20 0   20   37   40   60   80   100
°C

Unclassified

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20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is a state-of-the-art review of engineering approaches for the preliminary design and analysis of tension membrane structures. It defines three phases of construction which must be addressed, and then discusses the fundamental, nonlinear structural mechanics governing the behavior of each phase.

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Both computer and noncomputer methods are discussed in the report. The important engineering properties of fabric membranes are defined. The basic geometric nonlinearity of these structures is emphasized, and among other things, is said to preclude the practice of load case superposition.

Establishing the nonlinear static reference configuration will often remain a difficult task. Once established, however, a linear dynamic analysis can be conducted for purposes of preliminary design, relative to the static configuration. It is noted that the amount of prestress in the static configuration has a large affect on the vibration properties of the structure. Ultimately, nonlinear effects must be included in the final design and analysis. Current analysis methods necessarily involve the computer and are also expensive.

The report concludes with recommendations for research to reduce the cost of structural analysis, to obtain additional engineering data on fabric stress-strain laws, and to obtain experimental data on the behavior of tension structures.

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CHAPTER 1. INTRODUCTION

1.1 Purpose of Study

Tension structures are ones in which the main load-carrying members transmit applied loads to the foundations or other supporting structures by direct tensile stress without flexure or compression. Their cross-sectional dimensions and method of fabrication are such that their shear and flexural rigidities, as well as their buckling resistance, are negligible. There are two broad classes of tension structures: cable structures[10,12]* comprised of uniaxially stressed members and membrane structures[9,13,14] comprised of bi-axially stressed members.

The general class of cable structures can be further divided into four subclasses[12]: 1) single cables in which single cable segments, or several simply-connected segments, are subjected to loads predominantly in a single plane of action, e.g., suspension cables, tether or mooring lines, guy lines for towers or tents; 2) cable trusses in which prestressed segments are multiply-connected in a single plane and loaded in that same plane, e.g., cable-stayed bridges, double-layer cable-supported roofs; 3) cable nets in which prestressed segments are multiply-connected in a curved surface (synclastic or anticlastic) and loaded predominantly normal to that surface, e.g. hanging roofs, suspended nets; and 4) cable networks in which cable segments are multiply-connected to form a three-dimensional framework, e.g. suspension networks, trawl nets, multiple-leg mooring systems.

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* *Numbers in parentheses denote entires in the Bibliography.*

There are four subclasses of membrane structures[9]: 1) air-supported structures in which an enclosing membrane is supported by a small differential air (or fluid) pressure, e.g. stadia roofs, inflated temporary shelters or storehouses; 2) inflated structures in which highly pressurized tubes or dual-walled mats are used as structural members in a space structure, e.g. inflated beams, columns or arches, dual-walled shells, air cushion roofs; 3) prestressed membranes in which fabric or rubber-like sheets are stretched over rigid frameworks and columns to form enclosures or diaphragms, e.g. tents, masted roofs; and 4) hybrid systems in which membrane panels span between primary load-carrying members such as prestressed cables and rigid members, e.g. reinforced fabric roofs, fluid storage tanks.

The intent of this study is to review and evaluate solution techniques for tensile structures and to provide a current bibliography on applications, design and analysis of such structures. Methods for preliminary static and dynamic analysis and design are reviewed with the aim of identifying gaps in the body of knowledge concerning tension structures.

1.2 Motivation for Study

Tension structures are well-suited to support broadly distributed dead loads and live loads such as wind, ocean currents and wave drift forces. It should not be surprising that light-weight tension structures resemble biological forms, since they also support loads by tension in pneumatically prestressed skins and fibers. The history of early applications of cable and membrane structure as architectural forms and engineering systems is described in Refs. 1-12.

Some of the advantages of tension members for use as structural components are[9,14]:

1) they are light in weight and collapsible, implying ease in transportation and erection;

2) they are prefabricated in a factory, have low installation costs and are potentially relocatable;

3) for air-supported structures, the primary load-carrying mechanism is the habitable environment itself, i.e., a pressurized mixture of gases;

4) the environmental loads are efficiently carried by direct stress without bending; and

5) they are load-adaptive in that the members change geometry to better accomodate changes in load patterns and magnitudes.

Land-based applications[5] include temporary shelters and warehouses, tents, hanging roofs, and suspension systems. Sea-based applications[15] include moored vessels and buoys, trawl lines and nets, towed arrays, floating hospitals and other logistical support facilities, floating or submerged breakwaters or storage tanks, and tension-leg or catenary-leg platforms.

Hybrid tension structures could be used as initial shelters in the erection of structures in hostile environments, e.g. in cold regions or under the ocean surface. The shelters could later be stiffened from the inside to form more permanent facilities. An inflatable shell, possibly made of vinylcoated fabric or thin metallic film, could be submerged in packaged form with all airhoses and airlocks attached. Then with minimal subsurface labor, it could be pressurized to the desired shape. Depending on the material used in the construction of the shell and on the depth of its intended use, the inflated dome would provide either a semi-permanent environment or temporary falsework and shelter within which concrete could be sprayed to construct a permanent atmospheric environment. If a double-walled form were used, the pressurized air (or water) within the walls could be displaced by fiber-reinforced concrete pumped

from the surface.

Recent thought has been given to the feasibility of using pressurized membranes to roof small bases or towns, the membrane being supported primarily by the circulating interior air.

Because of their reduced stiffness characteristics, tension structures are susceptible to large motions due to concentrated loads and dynamic effects. They respond in a nonlinear fashion to both prestressing forces and in-service forces, irregardless of linearity of material or loads. Prestressing forces are those forces (edge loads, self-weight or pressure) which act on a predominant static equilibrium configuration of the structure. They stabilize the structure and provide stiffness against further load. The response of a tension structure to prestressing forces is always nonlinear in that the equilibrium config- urations, as well as the state of stress, are dependent on those forces.

In-service forces are those variable live loads, static or dynamic, which the structure may be expected to encounter during its service life. They are superposed upon the prestressing forces. The response to in-service forces may be nonlinear or quasi-linear depending on the directions and magnitudes of the in-service forces relative to the state of stress in, and configuration of, the prestressed structure. The response is not strictly linear and therefore superposition of results for different in-service loading conditions is not strictly valid and, if done, must be done carefully.

It is usually sufficient to consider only linear (possibly piece-wise linear) material behavior for tension structures[15]. There are instances, however, where nonlinear material characteristics should be considered: hyper- elastic and visco-elastic behavior of polymer cables[16] and membranes[17]; non-isotropic woven fabrics[19], thermal-elastic and elasto-plastic behavior under extreme load[20]. Another potential source of nonlinearities of response

is the interaction of tension structures with hydrostatic and hydrodynamic loads. Not only are the magnitudes of drag force nonlinear, but also they are non-conservative in that directions of pressure loads are dependent on orientations of the cable axes and membrane surfaces which may undergo considerable rotation during loading.[21]

Because of the inherently nonlinear nature of tension structures, conventional linear analysis which assumes small elastic deformations and displacements is often not applicable to cable and membrane structures. Over the past decade there has been considerable development of analysis techniques and computer codes for tension structures. Simplified procedures for preliminary designs are evolving. Relatively inexpensive techniques and computer codes must be selected for the preliminary sizing and evaluation of candidate design concepts. After an initial design has been selected and detailed, refined analysis procedures must be used to evaluate the nonlinear dynamic response of the structure to both expected and extreme load conditions.

1.3 Scope of Study

The fundamental mechanics of behavior of cable and membrane structures is covered in Chapter 2 wherein is discussed the nonlinear nature of tension structures, various assumptions and simplifications possible, and the behavior and selection of materials commonly used in tensile structures. In Chapter 3 are described preliminary analysis and design procedures for cable and membrane structures based principally on statical considerations. Various numerical procedures and computer codes are discussed and basic aspects of the dynamic analysis of tension structures are included. Recommendations for further study into the behavior of tension structures are given in Chapter 4. A current bibliography on applications, design and analysis of tension structures is included in the Appendix.

CHAPTER 2. MECHANICS OF TENSION STRUCTURES

In this chapter the fundamental mechanics of tensile structural behavior is considered with the objective of identifying similarities and dis-similarities of their response to that of conventional structural systems. The implications of strictly tensile behavior and of potentially large displacements on analysis and design techniques will be discussed. Finally, strength and other characteristics of materials commonly used in the fabrication of tensile structures will be summarized.

In every formulation of a continuum mechanics problem field equations are developed which describe the geometry, statics, kinematics and material behavior of the continuum and the external constraints on applied forces and displacements of the boundary of the continuum. This overview of tension structural behavior will proceed within the context of specifying the field equations of cables and membranes for each phase of their response during erection and service.

The physical behavior of a tensile structure during the application of loads can be divided into three primary phases[26]. The first phase is the deployment phase in which the cable or membrane system unfolds from its compact configuration into a state of incipient straining. The second phase is the prestressing phase in which the cable or membrane system deforms into a predominant equilibrium configuration under the action of dead weight, pressure or other fixed lifetime loads. The final, or in-service, phase is the stage in which the fully pre-stressed system is subjected to variable live or dynamic loads during its service life.

2.1 Deployment Phase

In the deployment phase the cable or membrane unfolds with external forces counter-balanced by inertial forces only. During this dynamic process the structure is stress-free and the statical and kinematical behavior is like that of a collection of rigid particles constrained by the topology of the fabricated cable or membrane elements. Using the notation of Schuerch and Schindler[21], we may summarize the analysis of this phase as follows: the middle surface and space curves S are deformed continuously into a set *S(p) of surfaces, where p is a parameter of the deformation. The curves and surfaces *S(p) are isometric with S, continuous wherever S is continuous and have the same topological properties as S.

The expansion from the compact configuration to the state of incipient straining is highly nonlinear and no unique solution has been shown to exist. *This phase is not of primary concern in that stresses are negligible.* One need only worry about "trapped" wrinkles or kinks leading to local tears or knots, about rates of deployment in relation to lack of stability under unexpected external loads, and about transient overstresses due to initial velocities and accelerations when the structure is fully deployed.

The behavior in the deployment phase is primarily a problem in mathematical topology. The field equations involved are those of differential geometry of isometric kinematics, and of boundary constraints. Equations of statics and constitutive relation do not enter the picture until the state of incipient straining is reached and the prestressing phase begins.

2.2 Prestressing Phase

The second phase in the erection of a tension structure is the prestressing phase wherein the structure undergoes displacement from its state of incipient

straining, hereafter referred to as the initial state, into a static equilibrium state dictated by the fabricated geometry of the structure and the prestressing forces. Since the displacements during this phase are large, this is a nonlinear problem but unique solutions for the stresses and displacements are possible [11,26].

2.2.1 <u>Geometry</u>.—Of major concern in the stress analysis of tension structures is the definition of a reference geometry for the stress calculations. It is tacitly assumed in most solution procedures that one can specify the geometry for which all such calculations are made. This is not a trivial specification for tension structures. The state for which stress calculations are desired is the prestressed state and that shape may be significantly different in dimension or configuration from the initial state. Thus, it is possible to delineate nonlinear solution procedure for the prestressing phase based on the selection of reference geometry[14].

One of two alternate specifications is possible:

- prescribed initial geometry
- prescribed prestressed geometry

The more straightforward alternative from the fabrication point of view is the prescription of initial geometry. This, however, leads to the more difficult stress analysis problem in that it is necessary by incremental or iterative procedures to determine the nonlinear solution for the prestressed geometry and the stresses thereon.

The second alternative enables a simpler stress analysis procedure. However, the initial shape required to obtain exactly that prescribed pre-stressed shape may de difficult or impractical to fabricate[24,25].

It is common practice in linear approximations to adopt a compromise between the two alternatives. For a given fabricable initial shape, a pre-stressed shape is assumed which has a regular geometry and thus can be easily handled in the stress analysis. With considerable experience in making an assumption and observing fabricated and prestressed structures, it is possible to approximate a desired prestressed geometry - at least visually[23]. The difficulties in this approach are that 1) an exact match does not occur and stress calculations are based on a slightly erroneous geometry; 2) for a non-standard shape there is no guarantee that the fabricated shape will lead to even a close approximation to the desired prestressed shape; and 3) there is a tendency to forget the load adaptive nature of tension structures and to tacitly assume a single geometry under all loading conditions[24,25].

The descriptions of the differential geometry of space curves and of surfaces is an important aspect of the mechanics of tension structures. Due to dead weight, cable segments are continuously curved: in the absence of other distributed loads they form catenary curves. Counter-stressed networks of cables are prestressed configurations which do not depend on external loads, e.g. weight, to maintain their shapes. The counter-stressing is provided by pre-stretching combinations of cables curved in opposition to each other: the cables for which the principle loads are directed away from the center of curvature are called sagging cables; the counter-stressing cables are called hogging cables and have an opposite curvature[5,10,12].

Three-dimensional curves are formed whenever distributed loads act out of the plane formed by the gravity vector and the chord connecting the end points of a cable segment. Since fluid loadings on cables are directed along the tangent, normal and binormal vectors at each point on the cable[21] and since the magnitudes of their loads are dependent on the orientation of those base

vectors relative to the direction of fluid velocity, rates of change of the base vectors with arc length need to be formulated using the Frenet formulas[27].

For membrane structures, surface curvature is described by the so-called Gaussian curvature $G^{(5,27)}$ which is the inverse product of the two principal radii of curvature of the surface. If G>0, the surface is called synclastic, elliptic, or positive-Gaussian, and the centers of curvature in the two principle directions lie on the same side of the surface. If G<0, the surface is called anti-clastic, hyperbolic, saddle-like or negative-Gaussian, and the centers of curvature lie on opposite sides of the surface. If G=0, the surface is called developable, parabolic or zero-Gaussian, and at least one radius of curvature is infinite.

Synclastic surfaces (e.g. spheres) require external load, e.g. pressure or dead-weight, to maintain the prestressed configuration. Anticlastic surfaces (e.g. hyperboloids) do not require external loads: the opposing curvature provides counter-stressing as for sagging/hogging cable combinations. Zero-Gaussian surfaces (e.g. cylinders) are called developable because they are the only ones that can form curved surfaces from fabricated flat surfaces without straining. Thus, they are the most common surfaces adopted for air-supported structures. More complicated and architecturally interesting air-supported surfaces are obtainable by joining segments of flat surfaces[6]. This introduces geometric discontinuities in the prestressed configuration and localized seaming stresses along joints.

2.2.2 <u>Statics</u>.— The prestressing phase is a static equilibrium problem in that the state of stress and shape due to a predominant static load is sought. If the reference geometry is specified on the initial configuration, the equilibrium equations are nonlinear since the loads and stresses act on the

prestressed configuration with as-yet-unknown locations, orientations and curvatures of line and area segments. Because of the assumption of negligible bending rigidity, transverse loads are balanced by gradients in curvature multiplied by the internal tensions. Thus iterative solutions are necessary to determine the stress state: assumed displacements lead to calculable stresses which lead in turn to new assumptions for displacements.

If the reference geometry is completely specified on the prestressed configuration, e.g. specified inflated shape or specified cable sag, the equilibrium equations will include only unknown stresses. In many cases, e.g. shells of revolution or singly-connected cable segments, this will be a statically determinate problem and the internal stress resultants can be determined without recourse to kinematic and constitutive equations. However in cases where closed cable loops are formed or where in-plane membrane shear is possible, the equations are statically indeterminate and displacements must be considered.

2.2.3 <u>Kinematics</u>. ─ Because of the extreme flexibility of tension structures (no bending resistance, small cross-sections) large displacements occur during the prestressing phase and nonlinear strain-displacement relations should be used. Strains will be small but relative rotations are large and thus second-order terms of displacement gradients are significant, i.e. line segment lengths may not change much but they do translate and rotate appreciably due to transverse loads.

There are two definitions of "strain" possible: 1) extension ratio = "engineering" strain = change in differential length divided by original length in the reference state; 2) "true" strain = metric strain = one half difference in squares of differential length divided by the square of the original length

in the reference state. The true strain is a more accurate representation of the kinematics but, since small strains are usually assumed, the extension ratio is most commonly used because of its clearer physical significance. The definition of original lengths in the reference state is significant since if an energy principle is used to determine the equations of statics the strain must be referred to the same state as the stress.

In cases where the prestressed configuration is the reference configuration, the kinematic and constitutive relations are used to determine the geometry of the initial state. This is an inverse problem[28], i.e. given a solution, what problem was solved? For example, if a sagged cable configuration is specified, the equations of statics are used in the form of a moment analogy[10] to determine segment tensions and the kinematic and constitutive equations are used to determine the unstressed lengths of the segments.

2.2.4 <u>Material Behavior</u>. During the prestressing phase semi-rigid translations and rotations of differential segments predominate over strain effects. Thus, it is often sufficient for preliminary design purposes to assume inextensible behavior[20,34-37] for most engineering materials unless the system is highly redundant. In those cases incremental procedures can be used and piece-wise linear elastic behavior can be assumed. It is only in the last stages of prestressing and in the subsequent in-service phase that constitutive equations play an important role. Discussions of highly elastic (rubber-like) and inelastic materials is deferred until a subsequent section of this chapter when characteristics of materials are considered.

2.2.5 <u>Constraints</u>. —Boundary conditions for the prestressing phase involve prescription of surface tractions due to external prestressing forces, of edge forces and of support motions. Dead weight loads on cables and membranes are conservative forces. Pressure loads from internal gases or from hydrostatic loads are non-conservative in that they change directions and magnitudes as the system of cables and membranes undergoes finite deformation. Thus, if the initial shape is taken as the reference configuration, the external force terms in the equilibrium equation will be displacement-dependent. If the prestressed shape is used as the reference configuration, the load nonlinearity will instead occur in the equations used to determine the initial shape.

Tension structures have negligible bending and buckling resistance. The boundary restraints must be consistent with that behavior. Clamped edges cannot be realized: instead, the cables and membranes will undergo localized kinking. Also, if the boundaries of a membrane structure were constrained such that one of the principle stresses would have to be compressive, the membrane would instead wrinkle transverse to the direction of the predicted compressive stress. Care must be taken in the design of boundary supports or restraining members such that wrinkling is minimized during the finite deflections that occur in the prestressing plane.

The lack of compressive strength of membranes also has implications in the selection of a desired configuration for the prestressed membrane. There are limitations on possible surface geometrics for air-supported structures in that there are certain shapes[5] which imply compressive stresses even under internal pressure and which therefore cannot be realized without extensive wrinkling.

2.3 In-Service Phase

The third phase in the behavior of a tension structure is the in-service phase wherein various static live loads, e.g. snow loads, and dynamic loads,

e.g. wind or wave loads, which are expected to occur during the service life of the structure are superposed on the prestressed configuration. Depending on the relative magnitudes of those loads compared to the prestresses, the behavior in this phase can be considered linear or nonlinear[40]. The prestressing stiffens the structure and the additional deflections due to in-service loads are considerably smaller than the prestressing displacements.

The geometry of the prestressed configuration can be used as the basis for stress calculations during the in-service phase in that usually only slight perturbations of that shape occur. If large excursions under added load are expected, then an incremental loading technique[38,39] can be used in which occasional recalculations of reference geometry and prestress are made before additional increments of load are considered. This same procedure can be used to predict a new prestressed configuration if there is significant change in the predominant loads on the tension structure.

The equations of statics or equations of motion for the in-service phase are linearized equations in that a fixed geometry is considered and additional displacements are small. Some nonlinear effects are included by accounting for the effect of prestressing on the membrane or cable stiffness, i.e. "geometric" stiffness contributions are considered[41]. In dynamic analyses of tensile structures it has been found that the magnitude of prestress has an effect not only on the frequencies of vibration but also on the mode shapes[42,43].

The behavior during the in-service phase is not in general statically determinate. The kinematical and constitutive equations must be incorporated into the equations of motion. Piece-wise linear material behavior is usually assumed. An important aspect in the analyses of in-service behavior is the calculation of reductions in tensile stresses due to added loads[44,45]. Slackening of cables and wrinkling of membranes should be avoided or, if

unavoidable, its extent and effect on global stability of the structure predicted. Flutter and dynamic wrinkling of cable-supported and air-supported roofs under wind loads have been shown to be important considerations in the design of such structures[46,47].

2.4 Materials of Construction

2.4.1 Cables[12,34,48,49].—Cables are important structural elements in tension structures. They have been made of steel, Kevlar (registered DuPont trademark; a synthetic aramid fiber), fiberglass, and polyester. However, structural strands and structural ropes are most commonly utilized as cables. A strand is an assembly of steel wires wrapped helically around a center wire in one or more symmetrical layers. A rope is composed of a plurality of strands wrapped helically around a core. Wire ropes are treated normally as elements that resist only tension in the longitudinal direction. Because of this, high breaking strength is a desirable property of the wire rope. There are, however, other properties just as important: 1) small size; 2) low weight; 3) long fatigue life; 4) corrosion and abrasion resistance; 5) flexibility, and 6) good stretch and rotational behavior. These properties depend on the rope manufacture and wire control.

Cables are commonly viewed as purely axial elements; however, the helical amouring wires induce a torque as the helical wires try to "unwind" during axial loading. In addition to stresses in the wires resulting from the axial force component, the wires are subjected to bending stresses which are difficult to evaluate because of relative movements of the individual strands in ropes. Consideration of the effects of induced torque or an externally applied torque may be a significant factor in designing cables. A torque-balanced cable is one designed to yield zero or very small amounts of rotation under load. It

eliminates the problems of torque and twisting under tension. It is also recognized that induced torque decreases the ultimate strength.

The cable materials typically used in modern construction exhibit linear stress-strain relationship over only a portion of their useable strength. Beyond the elastic limit, the proportional relationships do not hold. The breaking strength efficiency, the ratio of cable breaking strength to the sum of the individual wire breaking strength, is greater for ropes and strand lay, as would be expected of a more uniform stress distribution. For many kinds of strands, the breaking strength efficiency of a strand is reduced as the number of wires in the strand is increased. Since the outer wires in a multiple wire strand usually have larger angles of lay, the drop in efficiency could be attributed to a more divergent stress distribution. However, the breaking strength efficiency is also affected by other factors, especially elongation properties of the wire. A rope made of wires having limited total elongation characteristics will be less able to bear the overstressing arising from an unequal distribution of strains and consequently will develop a lower breaking strength efficiency than could be obtained with more ductile wire.

One of the important failure modes is fatigue failure, which has been blamed as the most common cause of mooring rope failure. Another is the susceptibility of stainless steel to pitting and crevice corrosion. However, a criterion for defining a fatigue failure in structural strand has not been established. Because of strength limitations, new wire rope materials are of continual interest.

2.4.2 Membranes[13,29,50-54]. Various materials can be used in the fabrication of a tension structure, such as hyperelastic (rubber-like) materials, fabrics, composites, etc. Hyperelastic materials[50] are highly

nonlinear in nature and can sustain large strains. Because of their low stiff-
ness and strength properties they have limited applications. Much of the previ-
ous work has centered around the air-supported membrane. An ideal membrane
material should have high tensile strength, be flexible and ultra-light weight,
economical in cost, and compatible to the structural configuration. It should
have a good dimensional stability, and should be highly resistant to the propa-
gation of minor accidental damage; and be capable of being seamed or jointed in a
simple, cheap, reliable and mechanically efficient manner. Finally, the material
should be resistant to all aspects of environmental degradation, and should be
fire resistant. Membrane materials, to date, have relatively low resistance to
cyclic folding and buckling.

Membranes are specified as having a specific width and weight per square
yard. Thicknesses may be derived from the densities of the materials. Typically
materials are purchased in rolls of 48" to 72" width. Weights are 4 to 50 oz/yd²
with derived thicknesses of 0.001" to 0.060".

Strengths of membranes are described in terms of strip tensile, grab ten-
sile, tongue tear, trapezoidal tear and adhesion. Various test methods as listed
in Table 2.1 are available to determine the behavior of membrane materials.

Because of the orthotropic nature of most membranes, properties are given
for different weave directions for fabrics and for different roll directions for
films. Typical values for these strengths according to FED Standard 191 are:

        strip tensile - 20 - 1000 lbs/in

        grab tensile - typically 30% higher than strip values

        tear strength - 10 - 200 lbs

        adhesion strength - 2 - 40 lbs/in

Strip tensile strength most closely correlates with the fabric's resistance
to tensile failure. Tear strengths are a measure of the fabric's resistance to

Table 2.1 - Identification of Fabric Test Standards

TEST	CODE PROMULGATING BODY/AUTHORITY				
	ASTM	BS	DIN	FED	JSI
Coated Weight	D-751-72	-	-	191-5041	L1079-76
Tongue Tear Strength	D-751-66	-	53-356	191-5134	L1079-76
Trapezoidal Tear Strength	D-2263-68	2782	53-356	191-5136	L1079-76
Grab Tensile Strength	D-751-73	3424	53-354	191-5100	L1068-64
Strip Tensile Strength	D-751-73	-	53-354	191-5102	L1084-64
Coating Adhesive Strength	D-751-73	-	50-161 53-537	191-5970	K-6328
Hydrostatic	D-751-73	-	-	191-5512	L1079-76
Flame	-	-	53-382 53-391	191-5903 191-5910	A-1322-66
Cold Cracking	-	-	-	191-5874	K-6328
Abrasion	D-1175-71	-	-	191-5304	L1079-76
Membrane Joint Strength	-	-	53-282 to 53-288	-	A-8952-66 M-7102-69
Reinforcement Strength	-	3530 4928	-	-	-

Notation:
    ASTM = American Society for Testing and Materials (USA)
    BS   = British Standard (UK)
    DIN = West German Industrial Standard
    FED = Federal Procurement Standard (USA)
    JSI = Japanese Standards Institute

abuse and propagation of failure. Adhesion strengths are a measure of the resistance to delamination.

Higher strengths are associated almost exclusively with the fiber-reinforced membranes. The coated fabrics can achieve higher tensile strengths than the laminates because they may be fabricated with fabrics of higher count (closer weave). Coated fabrics also enjoy better adhesion. Laminates are fabricated with scrims of an open weave to permit the "strike-thru" necessary for good film adhesion. This method of construction does result in relatively higher tear strengths as the individual fibers are more free to bunch to resist tearing.

Some properties of the most commonly used fabric materials are discussed below.

1) Nylon - strong fiber with good recovery properties; affected by moisture to a greater extent than polyester; affected by sunlight and oxidation; flammable.

2) Polyester - not quite as strong as nylon, and a little more expensive; unaffected by moisture and resistant to environmental degradation; higher modulus than nylon, and has a little better dimensional stability; flammable.

3) Glass - strong material with complete dimensional stability, and exceptional resistance to all forms of environmental degradation; poor abrasion resistance; translucent; flame resistant; sewn seams are not satisfactory.

4) Polypropylene - strong, outstanding resilience to chemical degradation; experimental materials have very high strength; adhesive problems reported; coating materials must have a low temperature cure.

The discussion listed above contains several explicit and implicit incompatibilities, and it is not possible at present to satisfy simultaneously all requirements in a single material. The best practical solution is a composite material, namely, a coated fabric in which each of the components is selected to fulfill a particular set of conditions, and the final material is much better than the sum of the components in terms of overall performance.

Properties of various coating materials are:

1) Vinyl - low cost material which can be given good mechanical and weathering properties by suitable choice of formulation; deterioration is caused by loss of plasticizer; can be made transparent at the cost of reduced service life; stiffens in cold environments; can be heat-sealed.

2) Hypalon - excellent resistance to acid, oxidation, ozone, heat and sunlight aging; good mechanical properties; excellent abrasion resistance; low water absorption; good color retention; low cost; poor low temperature resistance; requires cemented or seam joints.

3) Neoprene - very similar to Hypalon in resistance and weatherability; better tear resistance and adhesion to base fabric; low cost; requires cemented or seam joints.

4) Fluoroelastomer - good to excellent mechanical properties; outstanding resistance to all types of deterioration particularly at high temperatures; transparent; high cost.

5) Polyurethane - excellent mechanical properties and superior abrasion resistance; excellent adhesion to base fabric. Reasonable weather resistance; only fair acid and flame resistance; transparent; can be heat-sealed; medium cost.

Recently, Teflon-coated fiberglass has been successfully used in fabric roof designs[57,58] and shows promise of being a durable, high-strength material. The fabric is 1/8 in. thick, transluscent (allowing up to 65% outside light penetration), flame resistant, tearproof and waterproof. The fabric is coated with Teflon fluorocarbon resin and therefore the coated material is self-cleaning and a 20 year life expectancy is anticipated. The Teflon dispersion contains additives that improve flexibility, abrasion resistance and solar transmission.

It is generally acknowledged that the tensile behavior of a woven fabric is different under uniaxial and biaxial loading conditions, but it is not generally known that the difference can be very large. This phenomenon can lead to a variety of design problems in air-inflated structures.

A final fabric property that deserves mention is the shear behavior. Coated fabrics are very commonly skewed, or bowed: that is, the filling yarns are not perpendicular to the warp yarns. Under biaxial load a skewed fabric will distort by shearing and the resultant deformation of the structure may be undesirable.

A recent survey of failures of air-supported structures in Canada[55] indicates that the most common causes are tearing of the fabric and inadequate anchorage details in conjunction with high wind speeds. The tear strength of the material is considered an important property and a tear should not propagate if the membrane is damaged while subjected to the maximum stress. Another experimental results indicated that the possibility of large-scale shear failures should not be overlooked when an air-supported structure contains a hostile fire[56]. Another failure mode is joint-failure, known as the "zipper" effect.

3. PRELIMINARY DESIGN OF TENSION STRUCTURES

The design of structural systems is a cyclic process of 1) sizing and layout of components, 2) load estimation, 3) stress and deflection calculations, and 4) comparison of results to standards of performance. In the early stages of this cyclic process simplified methods of stress and deflection calculation are desirable because of the repetitive use of the methods to re-configure and re-size components and to estimate prestress stiffening for in-service loads. In the final stages of the cyclic process refined methods are needed to more accurately calculate stresses for the final comparison to standards of performance. In this chapter simplified methods of analysis of tensile structure are considered.

Tension structures exhibit nonlinear behavior and this must be recognized in any approximate simplified method developed for cable and membrane structures. For purposes of initial design only static loads need be considered, i.e., prestressing loads to maintain a design shape and predominant in-service loads distorting that shape. Nonlinearities of response should be considered in the prestressing phase but linearized equations could be used for the in-service phase because of prestress stiffening. Superposition of results for different loading conditions should not be used to form load combinations. Simplified models of linearized vibrations about the prestressed configuration should be used to predict potential resonances of the structure under expected in-service loads. Methods of analyses and design of singly-connected cable segments and cable networks will be considered separately from methods for reinforced and unreinforced membranes.

3.1 Cable Systems

There are no well-established standards of performance or codes of practice promulgated for cable systems. Design guidelines are given in Refs. (9,59,60). An ultimate load factor concept is recommended as a performance standard for design stresses. The AISI (59) recommends that the specified minimum breaking strength of each cable should not be less than any of the following load conditions:

$$T_U = 1.5 \ T_D + 3.0 \ T_L$$
$$= 2.5 \ (T_D + T_L)$$
$$= 2.0 \ (T_D + T_L + T_W)$$
$$= 2.0 \ T_E$$

where

T_U = ultimate cable tensions

T_D = cable tension due to dead loads

T_L = cable tension due to live loads

T_W = cable tension due to wind loads

T_E = cable tension due to erection loads

Note that sequence of loading is important in that superposition is not valid, i.e., T_L should be calculated from

$$T_L = T_{DL} - T_D$$

where

T_{DL} = cable tension due to combined dead and live loads. Tentative recommendations for standards of design for cable-stayed bridges are given in Refs. (75,76).

Cable systems may be singly-connected or multiply-connected. Singly-connected segments are joined one to the other in series between supports and thus allow for marching-type solutions in which solutions can be propagated from one

segment to the next without having to assemble complete sets of equations. Multiply-connected systems may have more than two segments incident at a connection and may contain closed loops: thus, highly redundant systems are possible.

3.1.1 Singly-Connected Segments.

First, consider a single segment or a collection of singly-connected segments spanning between two supports. The important design parameters are the horizontal component H of tension, the sag ratio f of midspan deflection to horizontal span and the cable rigidity EA, where A = metallic area of a cross-section and E = Young's modulus of elasticity. The dead weight of the segments provides "gravity" stiffness to the cable when additional in-service loads are applied.

If the distributed load, e.g., dead weight or fluid drag, on each segment is resolvable into a single direction, the segment will lie in a plane containing the two end points of the segment and the unit vector in the direction of loading. At every point along the cable the component H of tension perpendicular to the direction of loading will be constant. An analogy can be drawn (10) between H times y (the distance from the deflected profile to the chord connecting the end points and measured in the direction of loading) and the moment M due to the same load applied to an equivalent simply-supported beam with span equal to the distance (perpendicular to the direction of loading) between the two end points. See Fig. 3.1.

If H or the distance y at one intermediate point were known, the profile could be completely determined by the moment analogy and the tensions would be

$$T = H [1 + (dy/dx)^2]^{\frac{1}{2}}$$

The initial unstressed length could then be found from the pertinent constitutive equation, e.g., for an elastic material
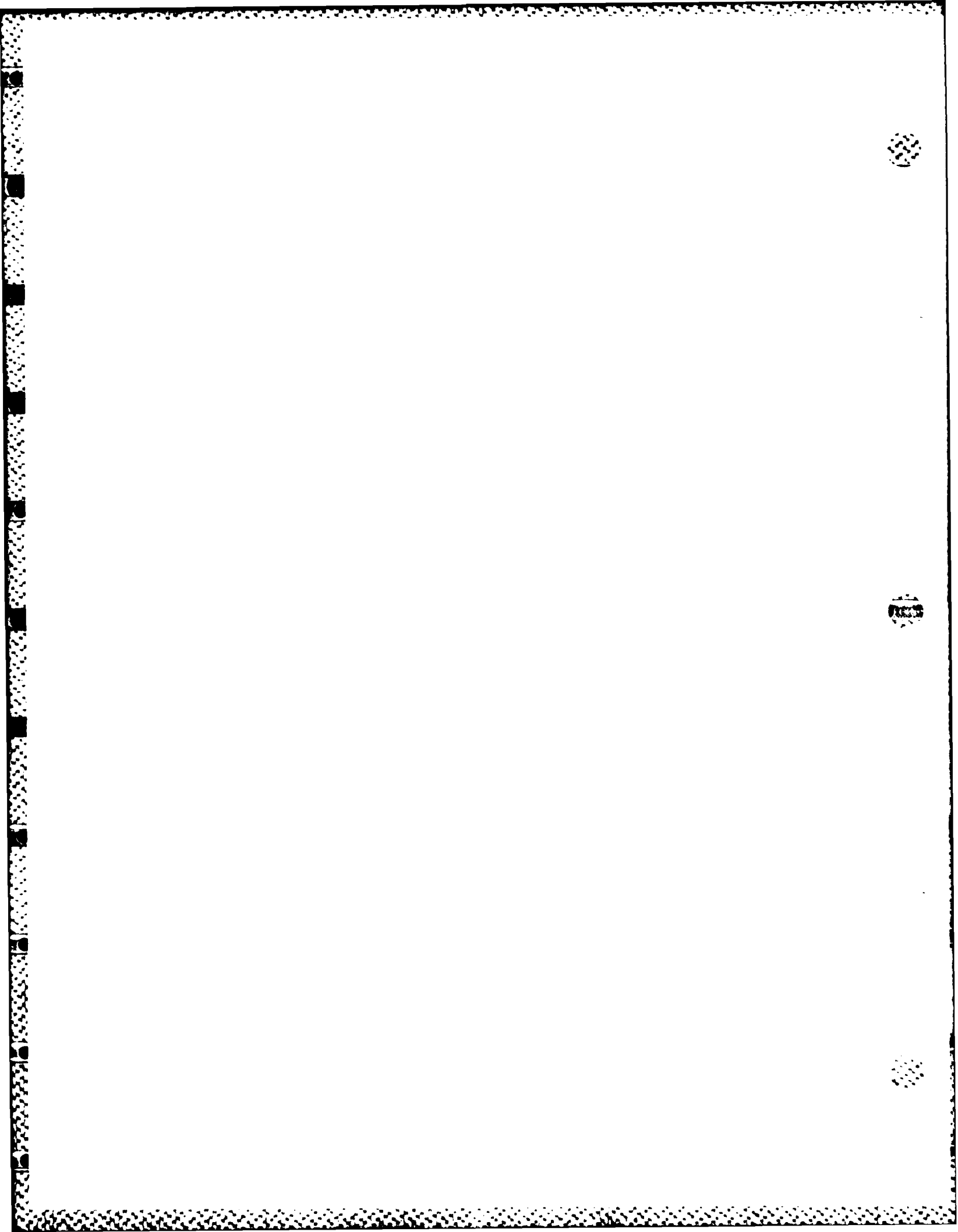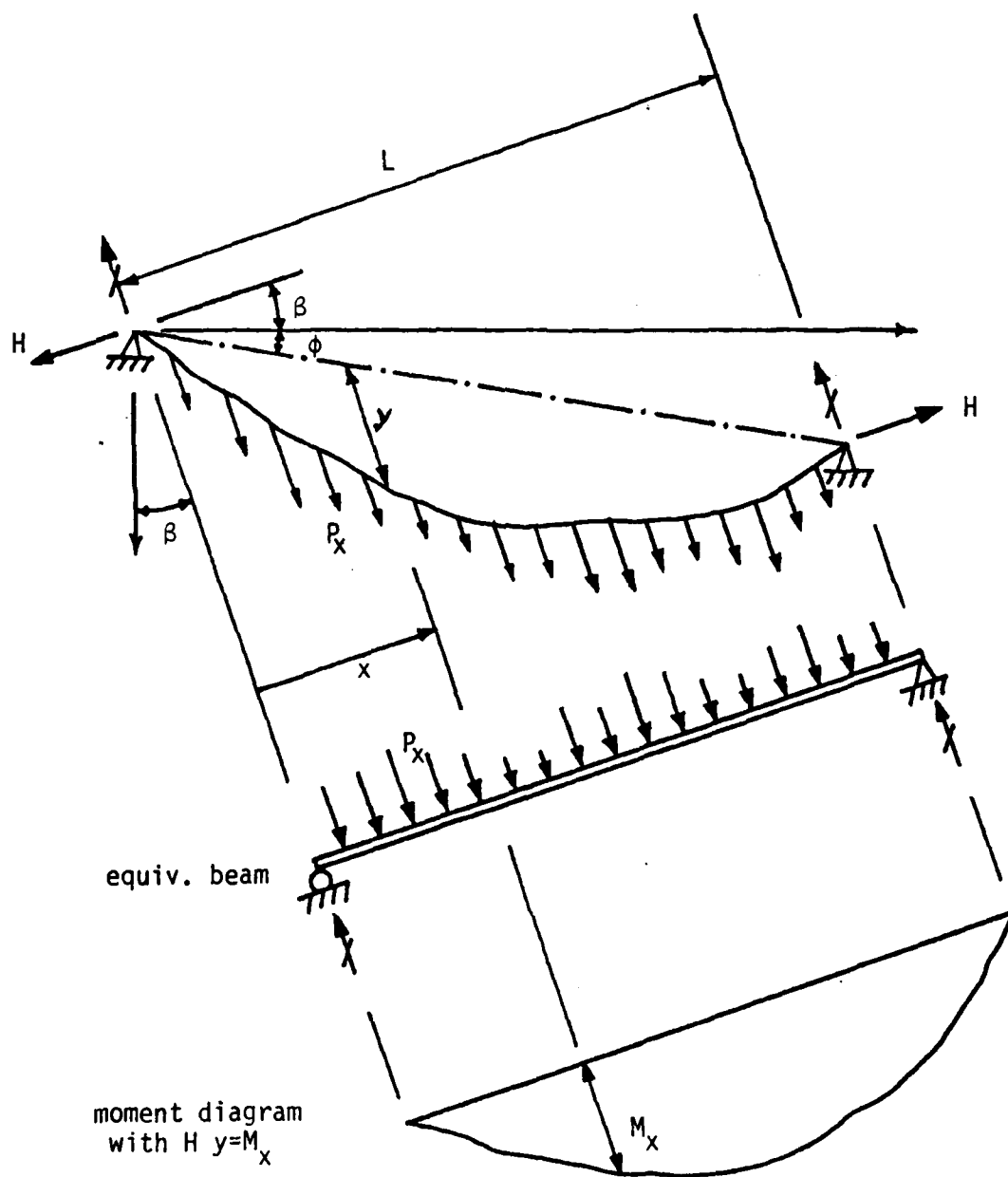
Figure 3.1 - Moment Analogy for Cable Segment

$$ds_0/ds = 1 - T/EA$$

where

ds_0 = increment of initial unstressed length

ds = increment of stressed length

$$= dx[1 + (dy/dx)^2]^{\frac{1}{2}}$$

This is often the procedure adopted in analyzing the prestressing phase of single cable segments in which a desired stressed configuration is prescribed. Degnerate examples of this case are the catenary solutions for uniform dead weight per unit arc length and the parabolic arc solution for uniform load per unit projected length (10,61).

If neither H nor any y are known an iterative solution is required with an additional condition required relating stressed and unstressed lengths to deflection y. This approach is often adopted for in-service phase problems. The parabolic arc solution has been found to be an adequate representation of single cable segment behavior for small sag ratios, i.e., $y_{max}/L \leq 0.2$. See Refs. 10,11,61-67 for further elaboration.

For several segments, either involving different cable properties, different directions or magnitudes of distributed load, or concentrated loads, iterative methods can be used in conjunction with a "shooting" method to integrate the cable equations from one support point to another. The behavior in each segment is specified as described above for a single segment (numerous straight segments with distributed loads replaced by equivalent concentrated loads or fewer catenary or parabolic arc segments). Equilibrium and continuity conditions at junctions between segments are used to propagate trial solutions from one segment to another in sequence from one support to the other. For trial values of "initial" conditions at the first support, closure errors in location or equilibrium at the final support are calculated. The closure errors

are then minimized by trial and error, usually in conjunction with a convergence acceleration gradient method such as the Newton-Raphson method (10,35,61,65,68).

In Refs. 62-64 are presented design expressions that can be used in an iterative solution to inclined singly-connected segments with various concentrated and distributed loads. The extensible parabolic arc approximation to the true catenary shape of each segment is used.

The method of imaginary reactions (69,70) is such an iterative shooting method applied to oceanic cables. See Refs. 71-74 for other treatments of hydrostatically loaded cable segments.

3.1.2 Multiply-Connected Segments.

For cable systems in which closed loops are formed, methods of redundant structural analysis should be used. If the segments lie in a single prestressed plane approximate methods based on a membrane analogy can be used (10,77,78). An equivalent membrane is defined with the same boundary topology as the cable net and continuous closed-form or open-form solutions applicable to transversely loaded flat membranes are generated. If the segments lie in a single curved plane momentless shell theory can be used to generate solutions to an equivalent curved membrane (26,38,79).

For non-planar systems the finite element method is recommended. Stiffness matrices for prestressed cable elements are presented in Refs. 20,30,43,80-85. In Refs. 39 and 43, the elements are assumed straight in the reference configurations and incremental methods can be used to calculate the prestressed shape as well as treat in-service loads. In Refs. 81,84, and 85 catenary elements are assumed. In Refs. 20 and 82 isoparametric approximations to three-dimensional curved elements are adopted.

Several commercially-available computer programs (86-87) contain in their library of elements prestressed truss-like cable elements for use in a

linearized analysis of in-service conditions: the prestressed configuration and stresses at some equilibrium state must be predetermined. Care must be exercised in cases in which a stable initial equilibrium state with significant stiffness in all degrees of freedom is not available. The first increments of loading must be extremely small (39) - no matter how small they are, some error will be introduced. For iteration schemes extremely accurate first-trial solutions must be generated.

The viscous relaxation technique (16,89,90) has been shown to be an efficient and accurate scheme for the treatment of problems without initial stiffness. In that technique a pseudodynamic problem with variable velocity-dependent damping is solved implicitly to determine a steady-state solution corresponding to the static nonlinear problem.

For initial design purposes in which either several different designs for the same prototype cable system are to be evaluated or several prototype systems with the same topology are to be evaluated, it may be very expensive and time consuming to employ a finite element computer program to model each design. In Refs. 91 and 92 is suggested a scheme in which a computer analysis of a scaled model with a reduced number of cable elements can be used to generate design charts. The charts can be applied to various prototypes with different cable properties as well as different numbers and spacings of cables. It is assumed that the material is linearly elastic, the loads are applied at the joints, the boundaries are much stiffer than the cable system, the system is uniformly loaded and prestressed, and the cables have uniform area and are uniformly spaced. The topology of the scale model and prototype must be the same, i.e., have the same boundary cable connectivity. Scaling is performed using Buckingham's Pi theorem to generate a generic network which is then analyzed to produce

design charts. The main advantage is in the reduced computational cost to analyze a network with a reduced number of cable elements.

3.1.3 Linearized Dynamics of Cable Systems.

If time varying forces shake a structure at less then one-third the lowest natural frequency of vibration for the structure, the response is essentially static in nature and dynamic motions and stresses can be considered negligible (93). If greater frequencies of time variation are present in the spectrum of external loads applied, then inertial forces on the structural components are mobilized and dynamic motions become important. For extremely high frequencies of load variation (greater than four times the highest natural frequency of vibration for the structure) there is insufficient time for the structure to respond to the load before the direction of load reverses and there are negligible dynamic motions due to those frequency components of the load (94).

In tension structures the stiffness is relatively small compared to other structural components and the mass may be large due to attached components or cladding. Since the natural frequencies are proportional to the square root of the stiffness to mass ratio, the natural frequencies of tension structures can be expected to be smaller than most other structural types. Dynamic responses to time-varying loads are more significant for tension structures in that dynamic stresses superposed on static prestresses may lead to failure of members due to overstressing or fatigue. The dynamic stresses are particularly large near resonant conditions, i.e., when the frequency of load is near a natural frequency of the system. In cable systems an increase in mass of the system, e.g., due to snow loads, may be accompanied by a decrease in stiffness of counter-stressing cables (stiffness is proportional to cable tension) and the natural frequencies may be significantly decreased. Thus, under combined load

systems, e.g., static snow load and dynamic wind loads, the dynamic response of the system may become more pronounced if the natural frequencies are decreased.

The dynamic loads that should be considered for cable structures include blast, seismic, wave and wind forces (10). Blast loadings due to diffraction and refraction about the structure of shock waves consist of an abrupt rise in pressure and suction which then rapidly decays. Seismic loads due to ground motions have a frequency range of 3 - 10 Hz. Cable structures have low natural frequencies (<4 Hz) whereas supporting structures have much higher natural frequencies. Since seismic loads are transferred to the cable system through the supports, high frequency components of the load will be amplified by the supports and the low frequency components attenuated. Thus, seismic efforts on the cable system may not be significant (10). Hydrodynamic wave pressures due to relative motion of the cables and the fluid have a frequency range from 0.05 to 0.5 Hz (96). Also, longer period drift forces and fluctuations in currents are significant on extremely compliant systems used as mooring systems (96). In a deterministic model of wind velocities loads due to wind pressures or suctions can be considered to consist of a slowly varying component plus a rapidly varying gust component with a frequency range of 0.1 to 1.0 Hz (97). It is low frequency gust components that are most important for cable structures. Because of the flexible nature of cable systems an aeroelastic model of their response to winds is preferable to a rigid structural model of the wind forces. Aerodynamic flutter may occur because of coupling between modes of vibration causing energy to be drawn from the airstream to excite instabilities.

Transient dynamic motions in structural systems eventually decay when the forcing functions are removed. This damping is due to conversions of mechanical energy into energy radiated from the structure by waves or heat. There are three sources of damping: material, connection and environmental. In addition

to the usual material hysterisis effects, material damping in cables is attributable to friction forces developed between the separate strands layed into a cable rope. Connection damping is associated with loss of energy due to friction in joints and support connections. Environmental damping is associated with interaction of the cable system with the medium in which it is embedded, e.g., aerodynamic damping, real fluid effects and radiation of waves in a fluid. Typical damping factors (10) for unclad cable systems are between 1/2 to 3% of a critical damping factor (value of damping at which no oscillation occurs and transient motion decays in a purely exponential fashion). When fluid damping in included, the damping factors range between 3 to 5% (11, 102). For cable systems with cladding, damping factors may become as high as 20% (98, 108). It has been found that damping is considerably greater for slack cables than for taut cables (11). Damping is a desirable aspect in that the peak dynamic stresses at resonant peaks of the dynamic response are considerably reduced by damping. It is very difficult to establish the form of damping in structural systems and to estimate percentages of damping present. Experimental work is required on models or prototypes to estimate the damping present in the system.

One factor to remember in the dynamic analysis of cable systems is that superposition of loads and displacements is not strictly valid for nonlinear problems. Thus, the modal superposition method of analysis of forced vibrations of linear systems in which results from decoupled analyses of the separate modes are superposed is not a true representation of the nonlinear behavior. However, if a prestressed static configuration is established which has significant stiffness and if small in-service dynamic loads are then added, it is reasonable to assume linearity of the superposed dynamic response. In that case, methods of linear structural dynamics can be used, as described in Refs. 93, 94, 106 and 110. The most common linear methods adopt the approach of transforming the

problem from the time domain to the frequency domain where harmonic analysis of each mode of natural vibration is performed and results appropriately recombined to obtain time histories of motions and stress. This approach is particularly useful in the analysis of stochastic loads which reflect the random nature of wind and real waves.

The vibrations of sagged cables and networks have been studied using the finite element method (43, 99, 100, 109) to determine parametric dependencies of frequencies and modes of vibration. Both the natural frequencies and mode shapes are dependent on the level of prestress. Natural frequencies increase with prestress and different mode shapes may develop. Curvature of the cable system has a strong effect on the natural frequencies (10,99). Increased curvature leads to increased frequencies.

Irvine (11) discusses the continuum behavior of vibrating cable segments. Wingert and Huston (101) present a technique for the dynamic analysis of inextensible singly-connected cable segments with hydrodynamic forces included. Iwan and Sergev (103) present a technique for rapid numerical evaluation of frequencies and mode shapes of singly-connected taut cables segments with lumped masses distributed along the cable.

For cable networks lying in a plane the membrane analogy can be used to predict free vibration behavior (104, 105). Several commercially available computer programs (86-88) include prestressed cable elements which can be used to predict linearized dynamics of three-dimensional networks. Nonlinear incremental and iterative methods for predicting the dynamic response of cable systems are described in Refs. 16,20,37,43,80,83,90,107.

3.2 Membrane Systems

Principles for the design of air-supported structures are set forth in Refs. 29,52,111-113. Various codes and voluntary design standards for air-supported structures have been promulgated (114-118). They vary widely in recommendations.

3.2.1 Static Analysis. It is common practice to use linear shell solutions for membrane structures assuming that the prestressed shape is the reference geometry for all calculations. If classical momentless theory is adopted, there are a multitude of solutions available in the literature. This is a practical approach if only stress calculations are desired and there is no need to calculate deflections.

In linear momentless theory for shells the stress can be decoupled from the deflection calculations. With proper boundary conditions the equilibrium equations are statically determinate and, depending on the shell geometry, can often be solved in closed-form without recourse to numerical simulation.

Momentless shell theory specially directed toward membrane structures is discussed in Ref. 6. A more lucid treatment of membrane theory is contained in Refs. 79 and 119. The elementary application to air-supported spherical and cylindrical shells is given in Ref. 111.

One broad class of shells for which closed-form membrane solutions can be derived are shells of revolution. The meridianal and hoop stresses due to internal pressure in a shell closed at the apex are (119)

$$\sigma_{mer} = \frac{pR_2}{2h}$$

$$\sigma_{hoop} = \frac{pR_2}{2h}\left(2 - \frac{R_2}{R_1}\right)$$

where p = pressure, h = thickness, R_2 = principal radius in the hoop direction,

R_1 = principal radius in the meridianal direction.

Linear momentless shell theory can be used to predict stresses due to in-service loads superposed on the prestressing stresses, e.g., wind induced stresses. Few analytical solutions are available and recourse to numerical simulations may be necessary. Standard shell programs can be used if the prestressed geometry is taken as the reference and only the superposed loads are applied. The stress results would then be added to those obtained for the prestressing phase. More accurate results would be obtained in finite element programs which include prestressed elements as part of their element library (86,87,88). The analyst is cautioned to recall that in reality the geometry of the membrane structure will tend to adapt itself to the superposed load. Contrary to the claim in Ref. 111, the results may not always be conservative.

Because of the relatively low design stresses allowable for fabrics common to membrane structures, it has been common to reinforce the fabric with regular-ly spaced cables (usually steel). The cables are considerably stiffer than the fabric and are hence the main load carrying members. They also restrain the deformations considerably.

Simplified analysis procedures for cable-reinforced membranes are discussed in Ref. 6 and 111. In another work (31) an iterative numerical technique is proposed for cable-reinforced membranes in which the membrane is constrained against an inextensible cable network. In the above listed works, the solution process consists essentially of transmitting known membrane edge tractions to cable segments at an assumed incidence angle and then determining the cable stresses and configuration under those loads. If the configuration calculated does not adequately match the configuration assumed for the transmission of membrane tractions to the cable segment, an iteration can be performed. The newly-calculated configuration (or an over-relaxed version) would serve as the new assumed configuration for transmission of loads from the membrane to the cable net.

If one accepts the premise that linear momentless shell theory can be used for the structural analysis of membranes, then several special approximations common to such theory can be invoked. For example, one could use shallow shell theory for translational shell geometries or beam analogies for cylindrical shell segments.

Shallow Shell Approximation. - For a segment of a membrane structure, perhaps spanning within a net of reinforcing cables, it is reasonable to assume that its rise to span ratio is relatively flat. In that instance one can approximate the surface coordinates as cartesian coordinates in the projected plane of the edges of the segment. Then certain simplification in the ex-pressions for the curvatures can be effected and equivalent loads on the planar projection could be obtained. The form of the resulting equations would be such that the Pucher stress function (79,119-121) could be used to generate solutions in terms of tractions on the edges of the segment. For cable-reinforced seg-ments, those edge tractions would be the loads transmitted to the cables. Other discussions of shallow shell analyses are given in Refs. 122-126.

Beam Analogies. - One class of air-supported structure is highly pres-surized cylindrical tubes used as beam/column segments to support external loads. See Refs. 5,111 for discussions of stress analysis techniques for such tubes. Applications of, and stress analyses of the nonlinear response of such inflated tubes are discussed in Refs. 130-133.

Within the linearized shell theory as applied to air-supported cylindrical tubes or segments, the beam analogy developed for momentless cyclindrical shells could be adopted. In that approach, equivalent moment and shear resultants on the total cross-section are defined in terms of the shear and direct axial stresses on the cylinder. Then, beam analysis procedures could be used to calculate those equivalent forces and hence the stress to be superposed on the

stresses due to internal pressure. This technique could be used even for non-circular cross-sections and for partial cross-sections. Analysis procedures developed for barrel roof shells could be used. The cable reinforcement on the edges of adjoining segments would be in a sense equivalent to edge beams between barrel roof segments. See Refs. 126-129 for alternative techniques for moment-less cylinder analysis.

3.2.2 Nonlinear Analysis Methods.

There are few nonlinear solution techniques for membrane structures that do not require numerical solutions on digital computers. The perturbation method developed by Leonard (26) can be used for hand-calculations of shell of revolution behavior during the prestress-ing phase. This method consists of superposing an infinite set of asymptotical-ly converging linear solutions to the original nonlinear equations for a mem-brane with the deformed surface used as the reference surface. The solution process is discussed in Ref. 26.

Trostel in Sec. 7.5 of Ref. 5 has considered an approximate nonlinear solution useful in the region adjoining the end closures of inflated circular tubes. That solution method can be used to account for the significant devia-tion of linear results for momentless shells from those predictable by exact theory at edges.

A bibliography, developed prior to 1973, of nonlinear methods relying on digital computer simulation is given in Ref. 14. More recent efforts directed specifically to membrane structures have relied on the use of finite element methodology. In Refs. 90,134-136 are described nonlinear finite element programs for cable-reinforced membranes in which curved cable and membrane elements are used. Ref. 134 addresses the problem of static behavior of elastic un-reinforced membranes during both the prestressing and in-service phase.

Refs. 135,136 address the problem of static behavior of rubber-like reinforced membranes. In both works it was found that there are instances where it is extremely difficult to establish a prestressed configuration if no consistent equilibrium shape with initial stiffness is available. Ref. 90 resolves this difficulty by using a viscous relaxation technique for membrane elements similar to that used in Ref. 89 for cable elements.

The programs developed in conjunction with the work described above require considerable computational effort on a large digital computer because of the need to solve large sets of nonlinear equations. Another approach described in Refs. 137-141 relies heavily on interactive computer graphics to provide an initial design. Again, finite element methods are used: in this instance straight cable and flat triangular membrane elements. The problem of specifying a prestressed configuration is addressed in Refs. 139,140. Several techniques are described for solving either for the referenced geometry given the pre-stresses or for the prestresses given the reference geometry, or a combination of stresses and geometry. Least square techniques and iteration of nonlinear algebraic equilibrium equations are two methods employed. In Ref. 141 in-ser-vice static problems are treated using nonlinear equations with an interesting inclusion of slip effects between membranes and reinforcing cables. In the above-described works sophisticated computer hardware is required (137,138) to implement the interactive computer graphics.

3.2.3 Dynamic Analysis. Initial designs are based primarily on static load conditions. Final designs should be checked under dynamic loading con-ditions. There are few computer programs which include large dynamic deflection of membrane elements (90,142).

The following material in this subsection on membrane dynamics is extracted from Ref. 95, written by N.F. Morris. Additional references have been provided and reference numbers are made consistent with those used in the rest of this report.

"Air-supported structures are basically either curved membranes, or curved membranes reinforced by steel cables or fabric webbing. The former type of structure is prevalent in the standardized air-supported enclosures, whereas the latter type is prevalent in custom-designed permanent low profile air-supported roofs. Because the membrane in such a roof is a lightweight fabric, the main load-carrying elements are the cables, and the structure can be analyzed as a cable net, neglecting the effects of the membrane entirely....

As stated previously, the dynamic model for a structure follows from the static model in that once one chooses a method for generating the [K] matrix, the mass matrix [M] can be generated with the aid of a lumped or consistent mass approach if the finite element method is used, or the Rayleigh-Ritz method if a continuous membrane approach is used. Therefore, in a sense, the equations of motion for air-supported structures can be derived in a straight-forward manner. There are methods available for their solution as well, although for nonlinear response the computer cost for obtaining a solution can be expensive (42,142,143). The problem, however, is evaluating the significance of the solution. As pointed out in (144), the problem in air-supported structures is not simply the solution for a vibrating membrane, the interaction of the structure with wind must be considered if an analysis is to be valid.

Research on the effect of wind on structures is a field which has grown substantially in the last two decades. Some general aspects of the field are presented in (145-149). Herein, only a qualitative description of the effect of wind will be attempted. As shall be seen not enough information on

air-supported structures is available for a quantitative discussion... Four general classes of behavior can be distinguished. The most familiar effect of wind on line structures is vortex shedding. It is well known that harmonic lift forces transverse to the direction of wind flow are caused by by the periodic formation of vortices at points of flow separation. The frequency of these forces is defined by the Strouhal number. A design criterion for slender structures is the avoidance of catastrophic vibration due to vortex shedding. This is achieved in practice by shaping the structure so that the natural frequency is not close to the Strouhal number, or if this cannot be done, by modifying the structure aerodynamically (150). It seems unlikely that vortex shedding can have a significant effect on large low three-dimensional systems, such as air-supported structures. Nevertheless, local vortices may be generated (145,148,149,151); laboratory studies have shown that conical vortex sheets, similar to those developed on delta wing aircraft, can form at straight overhanging edges on shell-type roofs. This phenomenon can be avoided by proper detailing of the leading edge of the roof.

Flutter is the second wind-induced problem for slender structures (152,153). Flutter occurs when the natural frequencies of a system in both torsion and flexure have almost the same magnitude. The effect of air flow can then induce a combined motion which leads to a catastrophic failure. Air-supported structures cannot fail in this manner. There is, however, a lesser known type of flutter which has occurred in airplane wings and may occur in air-supported structures, known as panel flutter (150). Panel flutter is a flow induced wrinkling of membrane surfaces. It usually occurs on wings in supersonic flow and is prevented by pressurizing the inner skin of airplane wings to develop high tensile stresses. Membrane panels in air-supported structures cannot be highly stressed in tension, so this form of flutter would

seem to be a definite possibility. Aside from (154), which is not concerned
with air-supported roofs, the writer has seen no study of this problem.

Galloping is the third class of phenomena which can be induced by air flow
over slender structures. It is the least understood of all aerodynamic motion
mechanisms and appears to be due to negative aerodynamic damping which cancels
out structural damping and leads to large dynamic response (155,156). Galloping
occurs in turbulent flow and is a definite possibility in air-supported struc-
tures. In fact, when one discusses roof flutter it is galloping which is under
consideration, not conventional flutter. Description of galloping requires the
use of nonlinear equations to describe the effect of velocity on the structure's
load. Therefore, a linear galloping analysis is not possible. Whether gallop-
ing can occur for an air-supported roof is an open question. No empirical
evidence is available on this topic, but some analytical work (45,46,157) has
indicated that it is a definite possibility. The argument presented in (157) is
difficult to follow, but the author does try to prove that wind-induced insta-
bility can occur in flexible roofs. No empirical coefficients were presented in
the paper and changes in mode shapes due to varying tensions in the roof were
neglected. If Kunieda's work (157) is correct, it should be possible to pass in
the limit to a method for generating pressure coefficients due to wind on a
roof. It is doubtful whether this actually can be done. It may be noted that
the effect of mode shape changes due to varying tension is significant for very
flexible roofs (42). Despite these criticisms, reference (157) is worthy of
study. If it is possible for roofs to gallop, then nonlinear dynamic analyses
are a necessity for design.

Most conventional structures are dimensioned such that none of the three
previously mentioned phenomena can occur. Nevertheless, dynamic pressures due
to wind may be significant. This is the fourth problem, the gusting effects due

to wind (158-160). Knowledge of these pressures is required for two reasons. First, the cable anchorages must be designed for the maximum load that can act on the roof. Dynamic pressures can be a significant portion of this load. Second, the membranes used for roofing panels can only tolerate a certain amount of deformation. Therefore, the relative distortion of the panel boundaries must be determined. Standard techniques have been developed for the gusting analysis of linear structures. Such methods can be applied to air-supported structures as well, but there are some problems involved in their application.

It is obvious that wind tunnel tests have been, and are still being, carried out on air-supported structures. Unfortunately most of this work is proprietary and has not been published. The only published work is (161). Therein are reported some tests on pneumatic structures such as domes and cylinders. The results are basically static, in that static pressure coefficients due to a uniform wind speed are presented. Dynamic effects were not considered, although it was reported that some cylinders oscillated extensively when wind flowed perpendicular to the longitudinal axis. No study of this phenomenon was carried out. This phenomenon has occurred in other wind tunnel tests also, but it is doubtful if it could occur in a large structure subjected to a nonuniform wind flow. The validity of wind tunnel testing for structures as flexible as air-supported structures is one which might be controversial if the results of wind tunnel tests were ever published. Proper instrumentation of such tests seems to be difficult because any device attached to the membrane alters the characteristics of the structure substantially. It must be kept in mind that air-supported structures are so flexible that their interaction with wind is quite difficult to model. Tests on the Tokyo Pavilion were discarded for this reason.

At present, little information on wind effects is available to a designer

of air-supported structures (162). He must rely on his own intuition. Consideration of galloping or flutter seems unrealistic at this time because so little analytical or experimental information is available. Even if flutter is possible, it is doubtful whether flutter would actually lead to failure, rather than just excessive vibration. The main structural dynamic problem would seem to be gusting because the cable anchorages must be designed to resist the added forces to turbulent wind motion....

There are two other aspects of air-supported dynamic behavior which should be mentioned. When such roofs are set into vibration under wind load, the air enclosed in the structure is also set into motion. Thus, there is an added mass effect which must be incorporated into the mass matrix. Some work on this topic is presented in (151,162). Neglect of this added-mass term is not conservative because any increase in mass will increase the possibility of a large dynamic response under wind load. In addition to the added-mass effect of the enclosed air, there is also an increase in damping due to aerodynamic damping (151)."

4. CONCLUSION

In this report techniques for the preliminary analysis of tension struc-tures were reviewed. A bibliography on applications, designs and analyses of cable and membrane structures was developed.

The fundamental mechanics of cable and membrane structures was considered from the viewpoint of developing equilibrium, constitutive, and kinematic equations, as well as boundary conditions, for each of the three primary phases of nonlinear geometric and material behavior. Characteristics of behavior duing each phase were used to make simplifying, but rational assumptions.

Methods of static analysis of cables and membrane structure which do not rely on usage of digital computers were described. Available computer programs which could be adopted or modified to perform statical analyses of tension structures were listed. For purposes of a preliminary design only methods of predicting linearized dynamic response of cable and membrane structures were presented along with a discussion of dynamic loading characteristics for such structures. Available computer programs for dynamic analysis were listed.

Tension structures are intrinsically nonlinear. If linearized methods of analyses are used this fact must be recognized: 1) results for different load systems should not be superposed; 2) the equilibrium geometry should be used in stress calculations; 3) momentless boundary conditions must be realizable. Since the structure is considerably stiffened during the prestressing phase, the dynamic in-service phase can be considered nearly linear - but the amount of prestressing will effect the natural frequencies and mode shapes of the struc-ture.

During the course of this study the following observations were made:

1) The state-of-the-art for analysis of cable systems is more advanced than for membrane structures. This is because of many more years of practical experience in the use of cables as a construction component and because of the uniaxial stress state in cables as opposed to the bi-axial stress state in membranes.

2) Dynamic response predictions are expensive for tension structures, particularly if nonlinear effects are included. Linear analysis is sufficient for purposes of initial analysis, but a nonlinear analysis is required for final validation of the structural design.

3) For many cable and membrane structures, it is difficult to predict an initial reference shape which is both in equilibrium and has sufficient stiffness to enable further incremental analysis. The viscous relaxation technique (16,89,90) is an effective scheme for establishing an equilibrium configuration in such cases.

4) There is very little experimental data for tension structures, particularly for membrane structures. There is practically no dynamic data for wind or ocean loaded tension structures.

5) Most analyses of membrane structures assume isotropic behavior but membranes are commonly made of fabric materials. Considerable data are needed on material properties for fabrics, on constitutive relations, and on numerical analysis models including orthotropy. Most finite element models can readily accommodate orthotropic materials.

6) There are no analysis techniques available to treat problems of wrinkling of membranes in cases where the material buckles in one of the principle stress directions instead of accepting compressive stress.

7) There are as yet no models available to treat the problem of the nonlinear interaction of large membrane structures with dynamic pressure fields from ocean waves. There are linearized interaction models under development.

8) For rapid analyses of cable systems, particularly with hydrodynamic loadings, in which several long sections of singly-connected segments are included, a non-finite element techniques similar to that developed for single sections (71,72) should be investigated.

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